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A tachyon lump in closed string field theory

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Abstract: We find a codimension one lump solution of closed bosonic string field theory. We consider vertices up to quartic order and include in the string field the tachyon, the ghost dilaton, and a metric fluctuation. While the tachyon profile clearly is that of a lump, we observe that the ghost dilaton is roughly constant in the direction transverse to the lump, equal to the value it takes in the nonperturbative tachyon vacuum. We explain, with a simple model, why this should be expected.

Keywords: Tachyon Condensation, Bosonic Strings, String Field Theory.

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1. Introduction

There is new evidence coming from closed string field theory (CSFT) [1], that there is a nonperturbative closed bosonic string vacuum. This tachyon vacuum was first found[1] by Yang and Zwiebach [3] by using covariant closed string field theory truncated to quartic order [4]. The calculation was subsequently improved by including more levels of massive fields [5], and the quintic vertex [6]. A computation of the effective potential of the tachyon and ghost dilaton by integrating out all massive fields [7] shows that the properties of the tachyon vacuum do not change much when the quintic term is included; this is evidence that truncating the CSFT action to a finite order is a meaningful approximation. The effective potential also shows that the tachyon vacuum is a saddle point; we will shortly comment later why this doesn't necessarily mean that it is unstable.

The nature of the tachyon vacuum remains mysterious, but it has been conjectured that it corresponds to the state of the universe after a big crunch ([8, 3]). To test this idea further, it is interesting to construct dynamical or solitonic solutions; this is partially motivated by evidence from p-adic string theory that indicates that closed tachyon lumps could be noncritical string theories [9]. In [10], Bergman and Razamat found that it is possible to find such lump solutions in the low-energy effective action of the tachyon, dilaton and metric. They found that the dilaton must grow far away from the lump, and that it has a constant gradient along it; this is a nice check that the lump corresponds to a noncritical string theory with linear dilaton background.

In this paper we use covariant closed string field theory to construct a lump depending on one spatial coordinate. While one may argue that two spatial coordinates are needed in order to construct a lump with a linear dilaton background along it, interestingly enough

[1] A nonperturbative vacuum was already found in [2] on the CSFT truncated to cubic order. To this order, however, the ghost dilaton doesn't participate, and interesting features of the vacuum thus do not appear.

we do find a lump solution with one codimension. Actually, in the last part of their paper [10], Bergman and Razamat already attempted to find a codimension one lump solution in CSFT. They found a lump whose field configuration sits in the tachyon vacuum far from the core of the lump, and reaches and overshoots the perturbative vacuum (i.e. vanishing tachyon and dilaton) on the core. We make a few criticisms on their calculation: They didn't include the metric perturbation which, as we will see later, must be included when we have non-homogeneous string fields; they considered only second derivatives of the tachyon field and neglected all the higher derivatives; and they also neglected the kinetic terms of the dilaton and massive fields. While the first two problems can be considered an approximation, the third one seems to be in contradiction with their result (their dilaton profile has an amplitude of the same order of magnitude as that of the the tachyon profile, as can be seen on their figure 5 (a)), and therefore unjustified. As we will see, the dilaton kinetic term, and more particularly its sign, will play an important role in our calculation. It is perhaps interesting to note that its magnitude on our solution is, however, rather small; it should become clear at the end of this section, that the dilaton must in fact avoid gaining kinetic energy.

It is instructive to consider a very simple (but still related to CSFT) toy model of one tachyon t (in this paper t denotes the tachyon, time will not appear in the equations) and one dilaton d given by the action

$$S = \int d^D x \left(-\frac{1}{2}\partial_\mu t \, \partial^\mu t + \frac{1}{2}\partial_\mu d \, \partial^\mu d - V(t,d) \right) . \tag{1.1}$$

Our convention for the metric is $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$. This is not much different from the calculation of Bergman and Razamat, except that we have been careful to take the right sign in front of the dilaton's kinetic term. The potential $V(t, d)$ has a local maximum at $(t, d) = (0, 0)$ where it is zero, a flat direction along the dilaton axis, and a saddle point at, say $(t_0, d_0) = (1, 1)$ where it is negative. Now let us suppose that the fields t and d depend on one spatial coordinate x. Let us start to look at the tachyon only, which has a regular kinetic term. Its equation of motion is

$$t''(x) = \frac{\partial V}{\partial t}. \tag{1.2}$$

This is simply the very well known fact that a solution $t(x)$ can be seen as the time-dependent trajectory of t in the inverted potential $-V$. Now let us look at the dilaton only, which has a kinetic term of the opposite sign. The equation of motion for $d(x)$ is

$$d''(x) = -\frac{\partial V}{\partial d}. \tag{1.3}$$

In other words, the profile $d(x)$ can be seen as the trajectory of d in the potential itself. Now let us consider both fields together, evolving in the potential $V(t, d)$. The equations of motion are

$$t''(x) = \frac{\partial V(t,d)}{\partial t}, \qquad d''(x) = -\frac{\partial V(t,d)}{\partial d}, \tag{1.4}$$

and the question is: can we understand the solution of this system of equations as the trajectory of (t, d) in some "pseudo-potential"? In other words, can we find $U(t, d)$ such that

$$t''(x) = -\frac{\partial U(t,d)}{\partial t}\,, \qquad d''(x) = -\frac{\partial U(t,d)}{\partial d} \quad ? \tag{1.5}$$

We can immediately answer this question negatively by noting that it implies

$$\frac{\partial U(t,d)}{\partial t} = -\frac{\partial V(t,d)}{\partial t} \qquad \text{and} \qquad \frac{\partial U(t,d)}{\partial d} = \frac{\partial V(t,d)}{\partial d}, \tag{1.6}$$

and considering the multiple derivative $\partial_t \partial_d U(t, d)$. From the first equation we obtain $\partial_t\partial_d U(t, d) = -\partial_t\partial_d V(t, d)$, while the second equation tells us that $\partial_t\partial_d U(t, d) = \partial_t\partial_d V(t, d)$. These two equations are clearly incompatible unless $\partial_t\partial_d V(t, d) = 0$. We conclude from this simple example, that it is hard to reach an intuitive understanding of the lump profile when the two fields' kinetic terms have different signs. But we nevertheless attempt a guess. In our situation, we know that the potential $V(t, d)$ at the origin, has a local maximum in the tachyon direction and is flat along the dilaton direction. In this special case where $\partial_d V(t, d) = 0$, we can obviously view the dynamics as happening in the inverted potential $-V(t, d)$. Next, we know that the potential has a saddle point at $(t_0, d_0) = (1, 1)$. We will assume that (maybe after some field redefinition) the tachyon corresponds to the "minimum direction", whereas the dilaton corresponds to the "maximum direction".[2] In terms of these redefined (and normalized) fields, the potential can be written locally as

$$V = \frac{1}{2}(t-1)^2 - \frac{1}{2}(d-1)^2 + \text{ terms of cubic order.} \tag{1.7}$$

In particular we have $\partial_t\partial_d V(t, d) = 0 +$ linear terms. And, to leading order, the equations of motion can be understood as the motion in the pseudo-potential

$$U = -\frac{1}{2}(t-1)^2 - \frac{1}{2}(d-1)^2, \tag{1.8}$$

a local maximum at $(1, 1)$. We will thus have a flat valley along the $t = 0$ axis and a local maximum at $(1, 1)$. We don't make any assumption about the rest of the pseudo-potential as it might not be defined globally. A function U with the above properties up to numerical factors, is given by

$$U(t,d) = \left(1 + e^{-(d-1)^2}\right)\left(\frac{1}{2}t^2 - \frac{1}{3}t^3\right). \tag{1.9}$$

On figure 1 we show two trajectories on this pseudo-potential. It is clear that a solution that starts from the tachyon vacuum $(t, d) = (1, 1)$ (here a local maximum) and goes through the perturbative vacuum $(t, d) = (0, 0)$, will continue forever towards the negative dilaton direction. A lump solution, however, should come back to the tachyon vacuum. Such a solution will necessarily pass through the point $(t, d) = (0, 1)$. In contrast with the result of [10], the lump does not go through the perturbative vacuum $(0, 0)$. This should not be surprising because the potential has a flat direction and we have, in fact, a family of perturbative vacua

[2]It is interesting to observe that, due to the unusual sign of the dilaton's kinetic term, the saddle point is actually *stable*!

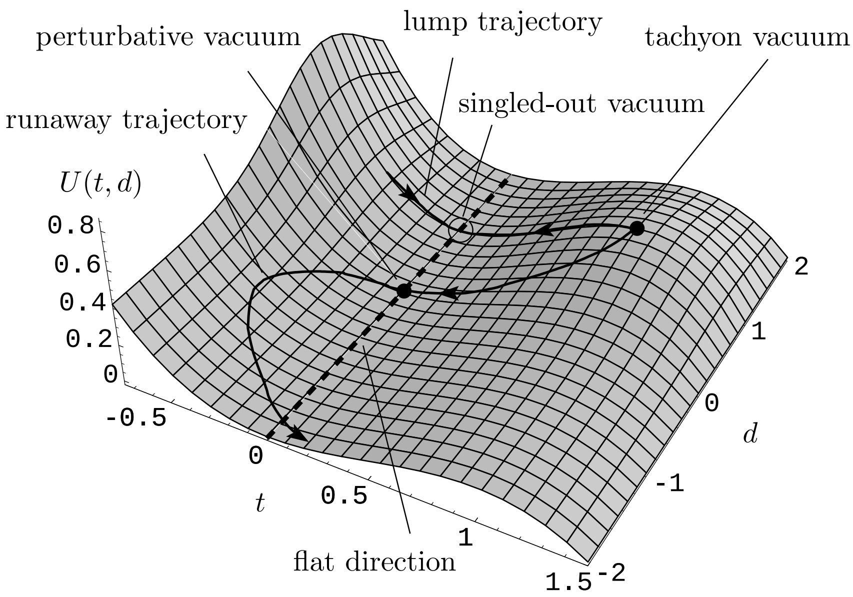

Figure 1: Two trajectories on the pseudo-potential: One that goes through the perturbative vacuum but then runs away, and a lump solution that comes back to its starting point, but does not go through the $(t, d) = (0, 0)$ perturbative vacuum, instead it "selects" another perturbative vacuum.

along this flat direction. By choosing $(t, d) = (0, 0)$ as perturbative vacuum, we are merely arbitrarily choosing one of them. It is of course well known that different vacua along the flat dilaton direction, are related by a change in the closed string coupling constant. After having made this basic observation, it should be clear that *there is no reason why the lump should go through the point* $(t, d) = (0, 0)$*, it could go through any other perturbative vacuum along the flat dilaton direction.*[3] From our simple model and figure 1, we may rather suggest that *the ghost dilaton will remain roughly constant.* In this paper, we will show that this is indeed what is happening in CSFT. Finally we take a slightly different point of view: Not knowing the tachyon vacuum, we would be unable to favor one perturbative vacuum from another along the flat dilaton direction (or in other words, a coupling constant from another). But now, the lump solution actually singles out one preferred perturbative vacuum, the one it crosses on its core. In other words, *the closed tachyon lump solution singles out one value of the coupling constant.* We will not speculate further along this line.

In the next section, we construct the codimension one lump solution in closed string field theory, and in the last section we will discuss its relation to the low-energy effective action.

[3] In the order- and level-truncated CSFT, the dilaton direction is only approximately flat and therefore only finitely many vacua exist nearby. Since $(t, d) = (0, 0)$ is a real extremum of the approximate potential, one may argue that we should still expect the lump to pass by this point. But the lump spends only a finite amount of "time" ("distance" would be more precise) near the flat direction, it is therefore not too sensible to the imperfections of our approximation; so, as we will see, the lump in fact does not go near the point $(0, 0)$.

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2. Tachyon lump in closed string field theory

We are considering the covariant closed string field theory action [1] truncated to quartic order [4]. And we use the convention $\alpha' = 2$.

$$S = -\frac{1}{\kappa^2}\left(\frac{1}{2}\langle\Psi|c_0^- Q_B|\Psi\rangle + \frac{1}{3!}\{\Psi,\Psi,\Psi\} + \frac{1}{4!}\{\Psi,\Psi,\Psi,\Psi\} + \cdots\right), \tag{2.1}$$

where $c_0^\pm = \frac{1}{2}(c_0 \pm \bar{c}_0)$ and Q_B is the BRST charge. We define the level of a state as the eigenvalue of $L_0 + \bar{L}_0 + 2$. The momentum thus contributes to the level; this is the obvious generalization of the level in open SFT which was shown in [11] to give rise to a well-convergent truncation scheme for states with momentum. The string field to massless level in the Siegel gauge is

$$|\Psi\rangle = \int \frac{d^{26}p}{(2\pi)^{26}}\left(t(p)\,|T;p\rangle - \frac{1}{2}h_{\mu\nu}(p)\,|H^{\mu\nu};p\rangle + d(p)\,|D;p\rangle\right), \tag{2.2}$$

where

$$|T;p\rangle = c_1\bar{c}_1|0;p\rangle\,, \quad |H^{\mu\nu};p\rangle = \alpha^\mu_{-1}\bar{\alpha}^\nu_{-1}c_1\bar{c}_1|0;p\rangle\,, \quad \text{and} \quad |D;p\rangle = (c_1c_{-1} - \bar{c}_1\bar{c}_{-1})|0;p\rangle \tag{2.3}$$

are respectively the tachyon, a metric fluctuation, and the ghost dilaton with momentum p. The quadratic term is easily calculated. With the convention

$$\langle p|c_{-1}\bar{c}_{-1}c_0^-c_0^+c_1\bar{c}_1|p'\rangle = (2\pi)^{26}\delta^{(26)}(p-p') \tag{2.4}$$

we find

$$\begin{aligned}\kappa^2 S^{(2)} &= -\frac{1}{2}\langle\Psi|c_0^-Q_B|\Psi\rangle \\ &= \int\frac{d^{26}p}{(2\pi)^{26}}\left(\left(1-\frac{p^2}{2}\right)t(p)t(-p) - \frac{p^2}{4}\,h_{\mu\nu}(p)h^{\mu\nu}(-p) + p^2\,d(p)d(-p)\right).\end{aligned} \tag{2.5}$$

It is important to note that, as discussed in the introduction, the kinetic term of the dilaton has the unusual sign. In the sigma-model as well, the dilaton comes with the irregular sign; however, after expressing the action in terms of the Einstein metric, the sign of the kinetic term of the dilaton becomes regular.

The cubic terms can already be a little bit complicated (see [2] for a complete expression), so we introduce from now on the assumption that only one spatial component, say the I component, of the momenta is nonzero (or equivalently that the solution depends only on the coordinate X^I). It follows that we need to keep only the component h_{II} of the metric fluctuation. Indeed the other components can be consistently set to zero because the terms $\{h_{\mu\nu}, h_{\mu I} : \mu,\nu \neq I\}$ always appear at least in pair in the action. We can thus write $h_{\mathrm{II}}(p) = h(p)$, which is the trace of the metric fluctuation, the *matter dilaton*. The

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cubic term is then (with $K = 3\sqrt{3}/4$)

$$\begin{aligned}
\kappa^2 S^{(3)} &= -\frac{1}{6}\{\Psi,\Psi,\Psi\} \\
&= -\frac{1}{6}\int \frac{d^{26}p_1}{(2\pi)^{26}}\frac{d^{26}p_2}{(2\pi)^{26}} K^{-p_1^2-p_2^2-(p_1+p_2)^2}\Big(2\,K^6\,t(p_1)t(p_2)t(-p_1-p_2) \\
&\qquad -\frac{3}{4}K^4(p_1-p_2)^2 t(p_1)t(p_2)h(-p_1-p_2) \\
&\qquad +\frac{3}{32}K^2\left(4-(p_1+2p_2)(2p_1+p_2)\right)^2 h(p_1)h(p_2)t(-p_1-p_2) \\
&\qquad -\frac{1}{32}\left((p_1+2p_2)(2p_1+p_2)(p_1-p_2)\right)^2 h(p_1)h(p_2)h(-p_1-p_2) \\
&\qquad -3K^2 d(p_1)d(p_2)t(-p_1-p_2)+\frac{3}{8}(p_1-p_2)^2 d(p_1)d(p_2)h(-p_1-p_2)\Big). \qquad (2.6)
\end{aligned}$$

2.1 Lone tachyon at cubic order

We start by truncating the CSFT action to cubic order. Since, to this order, the ghost dilaton always appears quadratically into the action, we can consistently ignore it. The matter dilaton h, however, can couple linearly to tachyons with nonzero momenta; we will thus first consider only the tachyon, then we will include the matter dilaton in the next subsection. From (2.5) and (2.6), we write the cubic action for the tachyon

$$\begin{aligned}
\kappa^2 S &= \frac{1}{2}\int \frac{d^{26}p}{(2\pi)^{26}}(2-p^2)\,t(p)t(-p) \\
&\quad -\frac{1}{3}\int \frac{d^{26}p_1}{(2\pi)^{26}}\frac{d^{26}p_2}{(2\pi)^{26}} K^{6-p_1^2-p_2^2-(p_1+p_2)^2} t(p_1)t(p_2)t(-p_1-p_2), \qquad (2.7)
\end{aligned}$$

The equation of motion then reads, after Fourier transforming back to position space

$$(2+\partial_x^2)K^{-2\partial_x^2}\,\tilde{t}(x) = K^6\,\tilde{t}(x)^2 \qquad \text{where} \qquad \tilde{t}(x) \equiv K^{\partial_x^2}\,t(x). \qquad (2.8)$$

At cubic order, the tachyon vacuum is at

$$t_0 = 2K^{-6} \approx 0.41620. \qquad (2.9)$$

In order to solve (2.8) for a lump, we now compactify the direction $x = X^I$ on a circle of radius $2\pi R$, and expand the solution $t(x)$ in an even Fourier series

$$t(x) = \sum_{n=0}^{N_t} t_n \cos(nx/R). \qquad (2.10)$$

Plugging this into Equ. (2.8), one obtains a system of nonlinear equations for the coefficients t_n that can be solved numerically. The numerical solution $t(x)$ will be meaningful if it converges in both limits $N_t \to \infty$ and $R \to \infty$. We do find a lump solution with these properties. We consider $R = 3$ and $N_t = 4$ (which is a consistent bound with respect to level truncation since here the highest tachyon mode t_4 has level 16/9. Had we included

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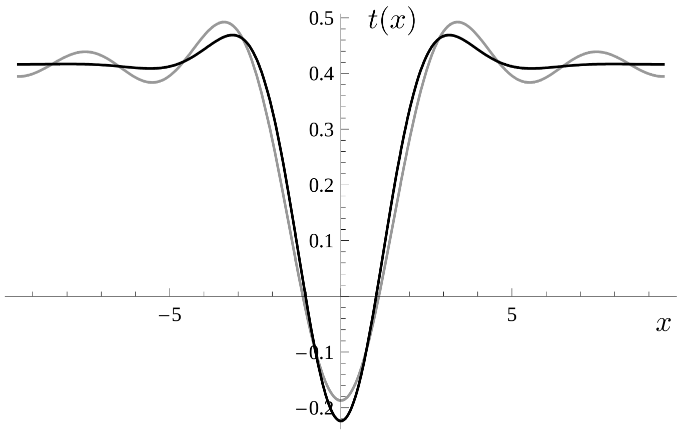

Figure 2: The pure cubic tachyon lump with $R = 3$, and $N_t = 4$ (gray curve) and $N_t = 10$ (black curve).

t_5, which has level 25/9, we should have also included the massless fields of level two in order to have a consistent level truncation). We find

$$t(x) = 0.33106 - 0.16679\ \cos(x/3) - 0.15448\ \cos(2x/3) - 0.12403\ \cos(3x/3) - 0.07283\ \cos(4x/3), \tag{2.11}$$

which is plotted with a gray line on figure 2. It is interesting to ignore level truncation consistency and add further harmonics to the tachyon. With $N_t = 10$ we find

$$\begin{aligned} t(x) = {} & 0.335385 - 0.159796\cos(x/3) - 0.151211\cos(2x/3) - 0.125671\cos(3x/3) \\ & - 0.077683\cos(4x/3) - 0.032177\cos(5x/3) - 0.009685\cos(6x/3) - 0.002342\cos(7x/3) \\ & - 0.000467\ \cos(8x/3) - 0.000076\ \cos(9x/3) - 0.000010\ \cos(10x/3), \end{aligned} \tag{2.12}$$

which is shown as a black curve on figure 2. It is interesting to observe that the tachyon profile oscillates with a damped amplitude as we move away from the core of the lump. This behavior is typical for a solution of an equation of the form (2.8) with a differential operator of the form $(2+\partial_x^2)K^{-2\partial_x^2}$; it was already observed in [12] in the context of tachyon kinks in cubic super SFT.

Now we would like to estimate the tension of this soliton. It is given by

$$\mathcal{T} = 2\pi R\,(V(\text{lump}) - V(\text{vacuum}))\,, \tag{2.13}$$

where $V(\text{vacuum})$ is the value of the potential evaluated at the nonperturbative tachyon vacuum. For a time-independent solution, the potential is minus the action, so (truncating to cubic order)

$$V = \frac{1}{\kappa^2}\left(\frac{1}{2}\langle\Psi|c_0^- Q_B|\Psi\rangle + \frac{1}{3!}\{\Psi,\Psi,\Psi\}\right). \tag{2.14}$$

With the solution (2.11) we find

$$\mathcal{T} = 0.23285\,\kappa^{-2}, \tag{2.15}$$

and we have checked that, to a good approximation, it doesn't depend on the compactification radius R. We have no physical interpretation for this number, but it is at least a good check that the lump solution is not a pure gauge transformation of the tachyon vacuum.

2.2 Including the metric fluctuation

The interesting thing now is to consider the effect of the lump on the gravity sector, namely the matter and ghost dilatons. In the next step, we stay at cubic order but go to level L with $2 \leq L < 4$, so that we must include the massless fields, but do not need to include massive fields. Since the cubic vertex can couple only an even number of ghost dilatons, we consistently set this field to zero for now. Instead of solving directly the equations of motion, as we did in the last section, we will write the action in terms of the tachyon and matter dilaton modes, and then find numerically an extremum of the action. The approaches are strictly equivalent and they lead to exactly the same numerical solutions, but the latter is a bit simpler when the equations of motion become complicated. Namely, we write

$$|\Psi\rangle = \sum_{n=0}^{N_t} t_n\, c_1\bar{c}_1\, \frac{1}{2}(|0;n/R\rangle + |0;-n/R\rangle) - \frac{1}{2}\sum_{n=0}^{N_h} h_n\, \alpha_{-1}^I\bar{\alpha}_{-1}^I c_1\bar{c}_1\, \frac{1}{2}(|0;n/R\rangle + |0;-n/R\rangle). \tag{2.16}$$

This is the same as (2.2) with

$$t(p) = (2\pi)^{26} \sum_{n=0}^{N_t} t_n\, \frac{1}{2}\left(\delta(p-n/R) + \delta(p+n/R)\right) \tag{2.17}$$

$$h(p) = (2\pi)^{26} \sum_{n=0}^{N_h} h_n\, \frac{1}{2}\left(\delta(p-n/R) + \delta(p+n/R)\right), \tag{2.18}$$

and with the normalization changed accordingly to the compact case

$$\langle 0;n/R|c_{-1}\bar{c}_{-1}c_0^- c_0^+ c_1\bar{c}_1|0;m/R\rangle = \delta_{n+m}. \tag{2.19}$$

So we can substitute (2.17) and (2.18) into (2.5) and (2.6) in order to write the action in terms of the modes t_n and h_n. We take again $R = 3$, and for the level of the fields to be lower than four (so we don't need to include massive fields), we must have $N_t^2/R^2 < 4$ and $N_h^2/R^2 < 2$, we will thus take $N_t = 5$ and $N_h = 4$. We find the following extremum of the action corresponding to a lump solution:

$$\begin{array}{lllll} t_0 = 0.35297 & t_1 = -0.12624 & t_2 = -0.12959 & t_3 = -0.12792 & t_4 = -0.09243 \\ t_5 = -0.03974 & & & & \\ h_0 = 0.07976 & h_1 = 0.11970 & h_2 = 0.04397 & h_3 = -0.00769 & h_4 = -0.01571 \end{array} \tag{2.20}$$

The profiles $t(x) = \sum_{n=0}^{N_t} t_n \cos(nx/R)$ and $h(x) = \sum_{n=0}^{N_h} h_n \cos(nx/R)$ are plotted on figure 3. We observe that the matter dilaton takes a roughly constant value of about $h \approx 0.22$ in the core of the lump, and zero far away from the lump. For the tension of the lump, we find

$$\mathcal{T} = 0.19166\, \kappa^{-2}, \tag{2.21}$$

not too different from (2.15).

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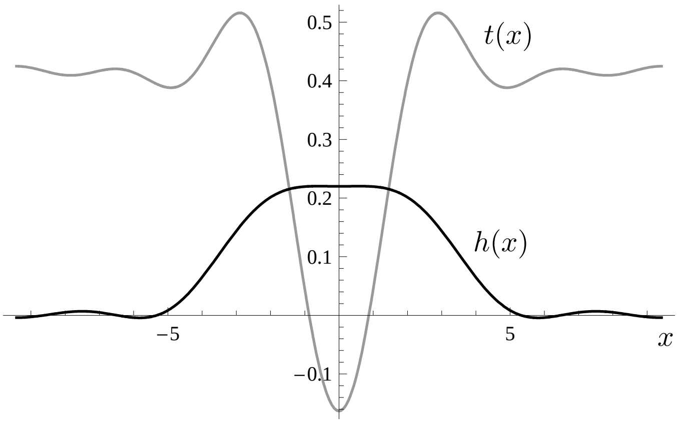

Figure 3: Tachyon (gray line) and matter dilaton (black line) profiles, with $R = 3$, $N_t = 5$ and $N_h = 4$.

2.3 Going to quartic order and including the ghost dilaton

We now include the quartic terms. For each term, we will need to perform an integration over the reduced moduli space of the quartic contact term $\mathcal{V}_{0,4}$. Details can be found in [13, 3−5]. Schematically, we have

$$\{\Psi_1, \Psi_2, \Psi_3, \Psi_4\} = -\frac{2}{\pi} \int_{\mathcal{V}_{0,4}} dxdy\, \langle \Psi_1, \Psi_2, \Psi_3, \Psi_4 \rangle_\xi, \tag{2.22}$$

where $\langle \Psi_1, \Psi_2, \Psi_3, \Psi_4 \rangle_\xi$ is calculated by 1) mapping the vertex operators Ψ_i from their local coordinates to a uniformizer on the sphere, 2) insert the antighost giving the measure on the moduli space, and 3) calculate the resulting correlator on the sphere Σ_ξ with punctures at 0, 1, ∞ and $\xi = x + y\, i$. For example, for four tachyons $|\Psi_i\rangle = c_1 \bar{c}_1 |0; p_i\rangle$, we have

$$\langle \Psi_1, \Psi_2, \Psi_3, \Psi_4 \rangle_\xi = (2\pi)^{26} \delta^{(26)} \left(\sum_{i=1}^{4} p_i \right) |\xi|^{2p_1 \cdot p_3} |1 - \xi|^{2p_2 \cdot p_3} \prod_{i=1}^{4} \rho_i(\xi, \bar{\xi})^{-2+p_i^2}, \tag{2.23}$$

where ρ_i are the mapping radii; their numerical expressions can be found in [4]. The important point is that the momentum dependence is highly nontrivial, it is not a simple function like $K^{-p_i^2}$ that we have in the cubic vertex; but it is an integral over reduced moduli space, parameterized by $\{p_1, p_2, p_3, p_4\}$. It would, in particular, be very hard to write an equation of motion as we did in section 2.1. Fortunately, in our setup of level truncation and compact coordinate X^I, we need only compute a finite number of terms. Our string field is now

$$\begin{aligned} |\Psi\rangle = &\sum_{n=0}^{N_t} t_n\, c_1 \bar{c}_1\, \frac{1}{2}(|0; n/R\rangle + |0; -n/R\rangle) - \frac{1}{2} \sum_{n=0}^{N_h} h_n\, \alpha_{-1}^I \bar{\alpha}_{-1}^I c_1 \bar{c}_1\, \frac{1}{2}(|0; n/R\rangle + |0; -n/R\rangle) \\ &+ \sum_{n=0}^{N_d} d_n\, (c_1 c_{-1} - \bar{c}_1 \bar{c}_{-1})\, \frac{1}{2}(|0; n/R\rangle + |0; -n/R\rangle). \end{aligned} \tag{2.24}$$

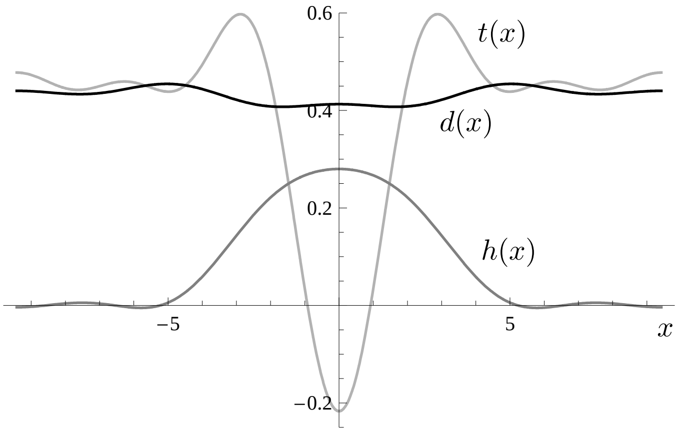

Figure 4: Tachyon (light gray line), matter dilaton (dark gray line) and ghost dilaton (black line) profiles, with $R = 3$, $N_t = 5$ and $N_h = 4$.

At quartic order without massive fields, the tachyon vacuum is at

$$(t_0, d_0) = (0.45933, 0.43878). \tag{2.25}$$

We take again $R = 3$. And we take $N_t = 5$ and $N_h = N_d = 4$, which are the maximal values allowed by consistency of level truncation. (With these values, the action contains 14 quadratic terms, 96 cubic terms, and 1183 quartic terms.) We find again an extremum of the action corresponding to a lump

$$\begin{array}{lllll} t_0 = 0.39264 & t_1 = -0.13780 & t_2 = -0.15585 & t_3 = -0.15875 & t_4 = -0.10640 \\ t_5 = -0.05067 & & & & \\ h_0 = 0.08881 & h_1 = 0.13897 & h_2 = 0.06093 & h_3 = 0.00281 & h_4 = -0.01164 \\ d_0 = 0.43151 & d_1 = -0.01523 & d_2 = -0.01343 & d_3 = 0.00156 & d_4 = 0.00841 \end{array} \tag{2.26}$$

The corresponding profiles are shown on figure 4. We note that the ghost dilaton is roughly constant along the lump, in agreement with the simplified model discussed in the introduction. To estimate the tension of the lump, we include the quartic term in the potential

$$V = \frac{1}{\kappa^2}\left(\frac{1}{2}\langle\Psi|c_0^- Q_B|\Psi\rangle + \frac{1}{3!}\{\Psi,\Psi,\Psi\} + \frac{1}{4!}\{\Psi,\Psi,\Psi\Psi\}\right). \tag{2.27}$$

And we find

$$\mathcal{T} = 0.25506\,\kappa^{-2}, \tag{2.28}$$

again not too far from the value (2.15).

3. Discussion

In [10], Bergman and Razamat studied closed tachyon lumps in the low-energy effective theory of the tachyon, dilaton and graviton

$$S = \frac{1}{2\kappa^2}\int d^D x\sqrt{-g}e^{-2\Phi}\left(R + 4(\partial_\mu\Phi)^2 - (\partial_\mu T)^2 - 2V(T)\right) \tag{3.1}$$

This action was first considered in [8]; its equations of motion are

$$R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \Phi - (\partial_\mu T)(\partial_\nu T) = 0 \tag{3.2}$$

$$\nabla^2 T - 2(\partial_\mu \Phi)(\partial^\mu T) - V'(T) = 0 \tag{3.3}$$

$$\nabla^2 \Phi - 2(\partial_\mu \Phi)^2 - V(T) = 0 \tag{3.4}$$

We shortly summarize the calculation of [10]. After making the ansatz $T = T(x^1)$, $\Phi = \Phi(x^1, x^2)$ and

$$ds^2 = (dx^1)^2 + a(x^1)^2 \eta_{\alpha\beta} dx^\alpha dx^\beta, \qquad \alpha, \beta = 0, 2, 3, \ldots D-1, \tag{3.5}$$

the equations of motion imply that the general form of $\Phi(x^1, x^2)$ must be

$$\Phi(x^1, x^2) = \mathcal{D}(x^1) + Qx^2 \tag{3.6}$$

for some constant Q and some function $\mathcal{D}$. And they also imply the constraint

$$Qa' = 0. \tag{3.7}$$

Bergman and Razamat considered $a' = 0$ and set $a = 1$, and thus found the equations of motion

$$2\mathcal{D}'' - (T')^2 = 0 \tag{3.8}$$

$$T'' - 2\mathcal{D}'T' - V'(T) = 0 \tag{3.9}$$

$$\mathcal{D}'' - 2(\mathcal{D}')^2 - 2Q^2 - V(T) = 0 \tag{3.10}$$

They found $Q \neq 0$, a linear dilaton along the x^2 direction. And interestingly, from a qualitative analysis of the equations of motion they could argue that $\mathcal{D}(x^1)$ must grow in both directions $x^1 \to \pm\infty$ away from the core of the lump. They concluded that this is consistent with a noncritical string theory in $D-1$ dimensions in a linear dilaton background. Looking back at (3.7), we can now understand that our lump corresponds to the other choice, namely $Q = 0$. Indeed we took from the beginning $\Phi = \Phi(x^1)$ and included the component h_{11} of the metric, so that a' need not be zero. Actually this is not quite so because our metric is of the form

$$ds^2 = b(x^1)^2 (dx^1)^2 + \eta_{\alpha\beta} dx^\alpha dx^\beta \,, \qquad b(x^1)^2 = 1 + h(x^1), \tag{3.11}$$

which is conformally related to (3.5); but it is trivial because in terms of

$$y(x^1) = \int_0^{x^1} b(\chi) d\chi, \tag{3.12}$$

it is simply $ds^2 = (dy)^2 + \eta_{\alpha\beta} dx^\alpha dx^\beta$. In this new coordinates the equations of motion become

$$2\frac{d^2\Phi}{dy^2} - \left(\frac{dT}{dy}\right)^2 = 0 \tag{3.13}$$

$$\frac{d^2T}{dy^2} - 2\frac{d\Phi}{dy}\frac{dT}{dy} - V'(T) = 0 \tag{3.14}$$

$$\frac{d^2\Phi}{dy^2} - 2\left(\frac{d\Phi}{dy}\right)^2 - V(T) = 0. \tag{3.15}$$

These are the same as (3.8)–(3.10) except that we lack a $-2Q^2$ in the left hand side of Equ. (3.15). It thus seems that we have an over-constrained set of equations and that we shouldn't find any solution; turning on one component of the metric was just trivial. On the other hand we can argue that our equations are not more over-constrained than (3.8)–(3.10) once we allow a nonzero cosmological constant because we find exactly the same equations after replacing $V(T)$ with $V(T) + 2Q^2$. The triviality of the metric corresponds to a gauge transformation in the CSFT side. Namely, it was shown in [3] that, to linear order in the fields, the gauge-invariants are

$$T = t \qquad \text{and} \qquad \Phi = d + \frac{h}{4}. \tag{3.16}$$

We can then choose a gauge where $p^\mu h_{\mu\nu} = 0$, meaning $h = 0$, and we have trivialized the metric perturbation. Equ. (3.16) also tells us the relation, to linear order, between the CSFT fields and the fields in the effective action. Looking again at figure 4, on can see that the linear combination $\Phi = d + \frac{h}{4}$ will stay roughly constant along the lump, and is not much different from d. An extension of this discussion would likely require the computation of higher order terms in the gauge transformation of the string field (see [14] for an interesting discussion of the relation between the fields); it is possible that these terms will make the dilaton profile less flat.

In conclusion, the physical meaning of our lump solution is not clear since it has no dilaton gradient along the lump, and maybe not even across it; but at least we know that it has a nonzero tension and that it is therefore not pure gauge. Actually, the constancy of the ghost dilaton along the lump was expected from the very simple model discussed in the introduction. It would be interesting to extend this calculation by considering a ghost dilaton depending on two codimensions x^1 and x^2 and see if it develops a gradient along x^2. The main difficulty seems to be that a linear dilaton cannot be expressed as a periodic function, so we cannot simply compactify x^2 as we did with x^1. At last, it might also be interesting to consider lightlike solutions [15].

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